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# The $\gamma\gamma \rightarrow \pi^0\pi^0$ and $\eta \rightarrow \pi^0\gamma\gamma$ Transitions in the Extended NJL Model

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## Abstract

We calculate within the Extended Nambu–Jona-Lasinio model the leading in  $1/N_c$  contribution to  $\gamma\gamma \rightarrow \pi^0\pi^0$  to all orders in the external momenta and quark masses. This result is then combined with the known two-loop Chiral Perturbation Theory and compared with the data and other calculations. A technical difficulty in the same calculation beyond order  $p^6$  for  $\eta \rightarrow \pi^0\gamma\gamma$  is identified and for this decay results up to order  $p^6$  are presented.

Dipion production in photon-photon collisions has been considered a good test of Chiral Perturbation Theory (CHPT) since its first calculations[1, 2]. For the neutral pion case, its leading contribution is order  $p^4$  in the chiral counting and the tree level vanishes at this order. The size of the prediction for the latter case was afterwards confirmed[3] but there was a discrepancy with the predicted behaviour as a function of the center-of-mass energy. This discrepancy could be understood within the context of final state scattering effects as was shown using several ways of unitarizing the lowest order CHPT amplitude[4, 5]. It has also been calculated within the framework of generalized CHPT[6].

This process ( $\gamma\gamma \rightarrow \pi^0\pi^0$ ) will be measured with precision equal to or better than the Crystal Ball data at DAΦNE and other  $\Phi$ -factories[7]. This prompted the calculation of the next-to-leading correction in CHPT. This is a two-loop calculation and was performed in [8]. The new free parameters appearing in the tree level contribution (order  $p^6$  in this case) need to be determined from other processes, this is at present impossible and leaves an uncertainty in the prediction from this calculation. Another possibility is to estimate them, here of course model dependence enters and we are leaving pure CHPT. In the original calculation[8], this was done using resonance exchange dominance in the same way as was done in [9] for the similar process  $\eta \rightarrow \pi^0\gamma\gamma$ , see [10, 11]. One of the problems appearing in this type of estimates is that the size and signs of several of the needed couplings of resonances are not well determined, still leaving an undesirable uncertainty to the prediction.

In the process  $\eta \rightarrow \pi^0\gamma\gamma$ , the loop contributions up to order  $p^6$  are suppressed by G-parity or the large kaon mass[9]. This was in fact confirmed in an explicit calculation of the one-loop[9] and part of the two-loop amplitude to order  $p^6$ [12]. The latter reference also contains a rather exhaustive discussion of the present theoretical status of this decay. Therefore, the tree level contribution to the amplitude is in fact the leading one and the above uncertainty becomes a dominant part of the uncertainty on the final result for this process. We will not treat this process in the same detail as  $\gamma\gamma \rightarrow \pi^0\pi^0$  for the reasons given below.

The prediction of higher order coefficients in CHPT from various models has some history. However, the simplest models are resonance exchange dominance, the constituent quark-loop model and the Extended Nambu–Jona-Lasinio (ENJL) model. It was found in [13] that the ENJL model[14, 15] does give a good representation of the order  $p^4$  coefficients, i.e. the so-called  $L_i$  coefficients[16]. In particular it improved on the description for the parameters in the explicit chiral symmetry breaking sector (i.e.,  $L_5$  and  $L_8$ ). For the corresponding predictions of resonance exchange dominance and the constituent quark-loop model see [17] and [18] and references therein, respectively. The constituent quark-loop prediction for  $\gamma\gamma \rightarrow \pi^0\pi^0$  was given in [11] and in [9] for  $\eta \rightarrow \pi^0\gamma\gamma$ . The calculation within the ENJL model of these processes to order  $p^6$  was also performed in two recent papers[19, 20]. There is some disagreement between them. In this paper we take the attitude that if the leading contributions start at rather high order in the

chiral expansion even higher orders might also contribute significantly. This is definitely the case for  $\eta \rightarrow \pi^0 \gamma \gamma$  [9, 12] where restriction to order  $p^6$  or making all order estimates significantly changes the results. In this Letter we therefore use the techniques of [21] to calculate the process  $\gamma \gamma \rightarrow \pi^0 \pi^0$  to all orders in the chiral expansion to leading order in  $1/N_c$  in the ENJL model. This is equivalent to calculating the tree-level contributions to all orders in the chiral expansion. The application of these results to the process  $\eta \rightarrow \pi^0 \gamma \gamma$  is also performed. Here  $N_c$  is the number of colours of the QCD group. It is at this level of approximation that this model has been phenomenologically tested. The Lagrangian of the ENJL model is given by

$$\begin{aligned} \mathcal{L}_{\text{ENJL}} = & \bar{q} \{ i \gamma^\mu (\partial_\mu - i v_\mu - i a_\mu \gamma_5) - (\mathcal{M} + s - i p \gamma_5) \} q \\ & + \frac{8\pi^2 G_S}{N_c \Lambda^2} \sum_{a,b} (\bar{q}_R^a q_L^b) (\bar{q}_L^b q_R^a) - \frac{8\pi^2 G_V}{N_c \Lambda^2} \sum_{a,b} [(\bar{q}_L^a \gamma^\mu q_L^b) (\bar{q}_L^b \gamma_\mu q_L^a) + (L \rightarrow R)] . \end{aligned} \quad (1)$$

Here summation over colour degrees of freedom is understood,  $a, b$  are flavour indices and we have used the following short-hand notations:  $\bar{q} \equiv (\bar{u}, \bar{d}, \bar{s})$ ;  $v_\mu$ ,  $a_\mu$ ,  $s$  and  $p$  are external vector, axial-vector, scalar and pseudoscalar field matrix sources in flavour space;  $\mathcal{M}$  is the current quark-mass matrix. For values of the input parameters we use the results of Fit 1 in [13]. These are  $G_S = 1.216$ ,  $G_V = 1.263$  and a cut-off  $\Lambda$  in the proper time regularization of 1.16 GeV. For the current quark-masses we use  $m_u = m_d = 3.2$  MeV and  $m_s = 83$  MeV. These are the values that give the physical neutral pion and kaon masses in this model. Other phenomenological consequences can be found in [13, 14, 15] and references therein.

The amplitude for  $\gamma(q_1) \gamma(q_2) \rightarrow \pi^0(p_1) \pi^0(p_2)$  can be written in terms of two amplitudes  $A(s, \nu)$  and  $B(s, \nu)$ . We use here the conventions of [8].

$$\begin{aligned} T(\gamma(q_1) \gamma(q_2) \rightarrow \pi^0(p_1) \pi^0(p_2)) = & \\ & e^2 A(s, \nu) [(q_1 \cdot q_2)(\epsilon_1 \cdot \epsilon_2) - (q_1 \cdot \epsilon_2)(q_2 \cdot \epsilon_1)] \\ & + e^2 4B(s, \nu) [(q_1 \cdot q_2)(\Delta \cdot \epsilon_1)(\Delta \cdot \epsilon_2) + (\Delta \cdot q_1)(\Delta \cdot q_2)(\epsilon_1 \cdot \epsilon_2) \\ & - (\Delta \cdot q_2)(q_1 \cdot \epsilon_2)(\Delta \cdot \epsilon_1) - (\Delta \cdot q_1)(q_2 \cdot \epsilon_1)(\Delta \cdot \epsilon_2)] \end{aligned} \quad (2)$$

where

$$s = (q_1 + q_2)^2 ; \quad t = (q_1 - p_1)^2 ; \quad u = (q_1 - p_2)^2 ; \quad \nu \equiv t - u ; \quad \Delta = p_1 - p_2 \quad (3)$$

and  $\epsilon_{1,2}(q_{1,2})$  are the polarization vectors of photons 1 and 2. For  $p_1^2 = p_2^2$  we have  $-2\Delta \cdot q_1 = 2\Delta \cdot q_2 = t - u$ . The above amplitude is manifestly gauge invariant. The cross-section in terms of  $A(s, \nu)$  and  $B(s, \nu)$  can be found in a simple form in [8], Section 2.

The calculation of the tree-level contributions at leading order in  $1/N_c$  to  $A(s, \nu)$  and  $B(s, \nu)$  in the ENJL model to all orders in the momentum expansion and quark masses is the main purpose of this paper. The method used is the same one used in [15, 21] to calculate several three-point functions. Here, we need  $SVV$ ,  $SPP$ , and  $VVP$  one-loop three-point functions, and  $PPVV$  and  $PVPV$  one-loop four-point functions and all possible full two-point functions. We refer to [15, 21] for notation and a detailed description of the method used. Essentially we calculate numerically the Green's function

$$\Pi_{\mu\nu}(q_2, p_1, p_2) = i^3 \int d^4x \int d^4y \int d^4z e^{i(-q_2 \cdot x + p_1 \cdot y + p_2 \cdot z)} \langle 0 | T(P(0)P(x)V_\mu(y)V_\nu(z)) | 0 \rangle \quad (4)$$

to leading order in  $1/N_c$ . Here  $V_\mu(x) = \sum_a Q_a (\bar{q}^a \gamma_\mu q^a)(x)$  is the electromagnetic quark current,  $Q_a$  is the flavour  $a$  quark electric charge in units of  $|e|$  and  $P(x) = i(\bar{q}^b \gamma_5 q^b)(x)$ . Then we form the correct flavour ( $b$  in  $P(x)$ ) combinations to obtain the pseudoscalar current that couples to the neutral pion. We calculate all the form-factors in  $\Pi_{\mu\nu}(q_2, p_1, p_2)$ , see Appendix A in [8] for their definition, and check that they satisfy Bose relations and gauge invariance in our numerical results explicitly. We then reduce the pion legs by going on-shell following the procedure described in [21]. The photon momenta are also taken at the mass-shell.

There are two main types of contributions: The first one is one-loop four-point function with a constituent quark in the loop connected to the outside legs via a chain of bubbles. With a bubble we mean a one-loop constituent quark two-point function, bubbles are then joined by ENJL four-quark vertices to built chains of bubbles. We refer to these contributions globally as just the four-point contribution. In a bosonized language this would be four-meson vertices (the one-loop four-point function) coupled to the external sources ( $V_\mu(x), P(y)$ ) by propagators (the chains of bubbles). The other contribution is, in bosonized language, diagrams with two three-meson vertices connected by a propagator and the remaining free legs connected to the external sources by propagators as before. These we refer to as three-point contributions and we label them by the spin-parity of the “meson” connecting the two vertices. For  $\gamma\gamma \rightarrow \pi^0\pi^0$  these are states with either  $I^G(J^{PC}) = 0^+(0^{++}), 0^+(2^{++})$ , and  $J^{PC} = 1^{--}, 1^{+-}$ , while for  $\eta \rightarrow \pi^0\gamma\gamma$  these are states with either  $I^G(J^{PC}) = 1^-(0^{++}), 0^+(2^{++})$ , and  $J^{PC} = 1^{--}, 1^{+-}$ . In the ENJL model we are using, only  $J^P = 0^+$  or  $1^-$  structures are present, other structures could be introduced, for instance, adding operators with extra derivatives in (1) like in [22] and/or Dirac structures. Consistently we use the value of the parameters obtained from a global fit to low-energy data within this model and thus we expect a good description with just these structures.

There are then, three non-vanishing contributions in this model: The four-point one, the three-point scalar one and the three-point vector one. The vector

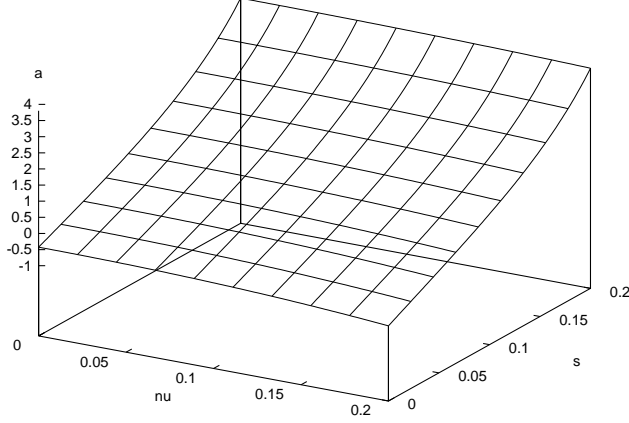


Figure 1: The form-factor  $a^r(s, \nu)$  in the ENJL model with the experimental value for the pion mass. The axis  $a^r(s, \nu)$ ,  $s$  and  $\nu$  are given in  $\text{GeV}^2$ .

three-point contribution is gauge-invariant by itself. The scalar-three point and the four-point contributions need to be added in order to be chiral and gauge invariant. E.g. at  $s = \nu = 0$  and for zero quark masses the amplitude  $A(s, \nu)$  should vanish. This is equivalent to say that the tree-level contributions in the chiral limit starts at order  $p^6$  for the neutral pion process. Our numerical result satisfies this which therefore provides a non-trivial numerical check on the calculation. To take out the expected order of magnitude of  $A(s, \nu)$  and  $B(s, \nu)$ , we define

$$a(s, \nu) \equiv \left(16\pi^2 f_\pi^2\right)^2 A(s, \nu), \quad b(s, \nu) \equiv \left(16\pi^2 f_\pi^2\right)^2 B(s, \nu). \quad (5)$$

From an analysis of the possible terms in the chiral Lagrangian, it follows that

$$a^{(6)}(s, \nu) = a_1 m_\pi^2 + a_2 s, \quad b^{(6)}(s, \nu) = b_1. \quad (6)$$

Deviations from this behaviour are an indication of the size of the corrections of counterterms beyond order  $p^6$  within the ENJL model. The couplings  $a_1$ ,  $a_2$  and  $b_1$  are order  $N_c^2$  in the large  $N_c$  counting. The amplitude  $a^r(s, \nu)$  for the case of real pion mass is shown in Fig. 1 and  $b^r(s, \nu)$  in Fig. 2. Where the superscript  $r$  means the corresponding finite regularized part. The Bose symmetry requires them to be symmetric exchanging  $\nu$  by  $-\nu$ . In both cases  $s$  and  $\nu$  vary between 0 and 0.2  $\text{GeV}^2$ . We have only plotted for this range of  $s$  and  $\nu$  since we have to stay away from the two constituent quark threshold where the artifacts of the ENJL model start dominating the results.

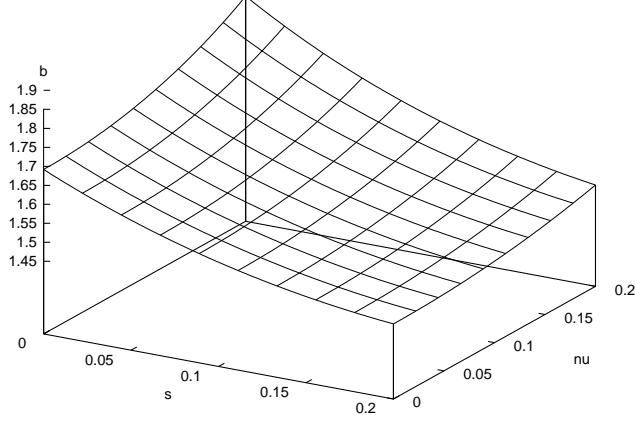


Figure 2: The dimensionless form-factor  $b^r(s, \nu)$  in the ENJL model with realistic quark masses. The axis  $s$  and  $\nu$  are given in  $\text{GeV}^2$ .

A good fit to the non-zero pion mass data displayed is given by:

$$\begin{aligned}
a^r(s, \nu) &= -0.03035 + 0.3773 s - 0.8218 s^2 - 0.07967 \nu^2 + \frac{1.626 s}{0.3140 - s} \\
&+ 0.4307 \left[ \frac{3 s + 2 \nu - 0.1458}{0.3182 + 0.5(s + \nu)} + \frac{3 s - 2 \nu - 0.1458}{0.3182 + 0.5(s - \nu)} \right], \\
b^r(s, \nu) &= 0.2723 + 0.1604 s + 1.798 s^2 + 0.0983 \nu^2 \\
&+ 0.2258 \left[ \frac{1}{0.3182 + 0.5(s + \nu)} + \frac{1}{0.3182 + 0.5(s - \nu)} \right].
\end{aligned} \tag{7}$$

For the case with zero quark masses a similarly good fit is:

$$\begin{aligned}
a_\chi^r(s, \nu) &= 0.0018 + 0.0919 s + 2.380 s^2 + 0.13512 \nu^2 + \frac{1.513 s}{0.2810 - s} \\
&+ 0.47527 \left[ \frac{3 s + 2 \nu}{0.3364 + 0.5(s + \nu)} + \frac{3 s - 2 \nu}{0.3364 + 0.5(s - \nu)} \right], \\
b_\chi^r(s, \nu) &= 0.08432 + 0.3323 s + 2.059 s^2 + 0.0012 \nu^2 \\
&+ 0.26426 \left[ \frac{1}{0.3364 + 0.5(s + \nu)} + \frac{1}{0.3364 + 0.5(s - \nu)} \right].
\end{aligned} \tag{8}$$

The constraint  $a_\chi^r(0, 0) = 0$  is satisfied by our numerics to about  $10^{-4}$ . That (8) deviates by a little more is due to the quality of the fit. A good fit to the vector

contribution alone in the case of non-zero pion mass is

$$\begin{aligned}
a_V^r(s, \nu) &= 0.08149 - 1.671 s + 0.7348 s^2 + 0.5022 \nu^2 \\
&+ 0.3333 \left[ \frac{3s + 2\nu - 0.1458}{0.3182 + 0.5(s + \nu)} + \frac{3s - 2\nu - 0.1458}{0.3182 + 0.5(s - \nu)} \right], \\
b_V^r(s, \nu) &= -0.3545 + 0.2566 s - 0.1292 s^2 - 0.1371 \nu^2 \\
&+ 0.1795 \left[ \frac{1}{0.3182 + 0.5(s + \nu)} + \frac{1}{0.3182 + 0.5(s - \nu)} \right]. \quad (9)
\end{aligned}$$

In all the fits above the form-factor  $a^r(s, \nu)$  is in  $\text{GeV}^2$  and  $b^r(s, \nu)$  is dimensionless. In these results we have used consistently the large  $N_c$  ENJL values for  $f_\pi$ , i.e. in the chiral limit  $f_\pi = 88.9$  MeV and for the non-zero quark-masses  $f_\pi = 90.0$  MeV. It should be remarked that we have chosen a type of meson dominance form to do the fitting but the values of the poles have no physical meaning. The expressions just provide a good fit within the kinematical regime mentioned. The scalar three-point contribution only contributes to  $A(s, \nu)$ , not to  $B(s, \nu)$ . The typical size of the vector contribution is about half of the total size for both  $A(s, \nu)$  and  $B(\nu)$  for small  $s$  and  $\nu$ . From the fits in the chiral limit we can extract  $a_2^r$  and  $b_1^r$ , from the finite quark mass result we extract  $a_1^r$ . The results are in Table 1. The second column is our ENJL result. The third column is the

	ENJL (this work)	Resonance Exchange ENJL[19]	Resonance Exchange Experiment	Vector Contribution ENJL	ENJL [20]
$a_1^r$	-23.3	-20.2	$-37.5 \pm 4.5 \pm 4.1$	-12.3	-12.1
$a_2^r$	14.0	12.3	$14.4 \pm 2.7 \pm 1.0$	4.4	10.3
$b_1^r$	1.66	1.30	$3.1 \pm 0.24 \pm 0.5$	0.73	0.97

Table 1: The comparison of the order  $p^6$  part of our result with meson dominance and existing ENJL model calculations.

resonance exchange dominance prediction in Ref. [19] within the ENJL model. In this reference, this result is added to the constituent quark-loop four-point function contribution. We believe this is inconsistent and this procedure is actually adding contributions from two different models: resonance exchange dominance and the quark-loop model. In fact, the four-point function alone does not fulfill chiral symmetry as said before while the resonance exchange contribution does by construction. The comparison of the second and the third column shows that the resonance exchange dominance works in the ENJL to order  $p^6$  within 15~25 % similarly to what happened to other quantities at order  $p^4$ [13]. The four-point and the three-point contributions combine to do this rather well. This is quite

important, since contrary to the order  $p^4$  [17] this is not well established yet and our result can be used as support for the use of resonance exchange dominance to this order as well. In the fourth column we show the result of resonance exchange dominance using experimental inputs. They include, of course, higher than order  $p^6$  corrections due for instance to quark masses and next to leading in  $1/N_c$  corrections. Here we have consistently used the experimental value  $f_\pi = 92.4$  MeV. For the contribution of the states with  $I^G(J^{PC}) = 0^+(0^{++})$  we have taken the signs favoured by phenomenology, see [12], and predicted also by the ENJL model [19]. The first error shown is the one from the input values and the second is the contribution from the states whose sign is not well established; i.e.  $I^G(J^{PC}) = 0^+(2^{++})$  [8]. As can be seen from the table only the ENJL result for  $a_2^*$  is compatible with the corresponding resonance exchange result. For the other two, although they only differ by one or two  $\sigma$ s, the central values are not quite compatible. The discrepancy is mainly because the decay rates for  $\rho, \omega \rightarrow \pi^0 \gamma$  are not well reproduced by the ENJL model[23]. The ENJL model does reproduce  $\rho^+ \rightarrow \pi^+ \gamma$  decay rate well[23]. At present, these reported radiative vector meson decays do not agree with nonet symmetry and therefore this discrepancy is not solvable within our approach. In the fifth column we show the contribution of the vector three-point function type of contributions to the results in the second column. In the sixth column we show the results obtained in Ref. [20]. We disagree with [20] but a look at the table makes it clear that in that reference the contribution from the vector part (our fifth column) was neglected. In view of the importance of this contribution, the approximation used there is not valid.

In Fig. 3 we have plotted for the cross-section for  $\gamma\gamma \rightarrow \pi^0\pi^0$  the one-loop result, the two-loop result with all the order  $p^6$  counterterms set to zero and the two-loop result with the full ENJL contribution added. The difference between the last two curves show the effect of the counterterms. As a comparison we have also plotted the result with only the order  $p^6$  part of the ENJL result plus the two-loop result. For the two-loop result we have used the simplified formula as given in [8] with  $\Delta_A = \Delta_B = 0$ . We have also indicated the presently available data[3] in this figure. The integration over the azimuthal angle was done up to  $|\cos\theta| \leq 0.8$  in this figure. From the formulas given above and the expressions in [8], the extension to the full integration range can be done easily. As can also be seen the total effect of the order  $p^6$  is quite small and the effect of the order  $p^8$  and higher orders is extremely small. So up to the energies shown, only a crude estimate of the extra counterterms is sufficient for this process.

The other decay  $\eta \rightarrow \pi^0 \gamma \gamma$  is more difficult to treat to all orders in momenta. The problem is that our whole approach is leading in  $1/N_c$ . The pseudoscalar mass eigenstates there, do not correspond to the physical  $\eta$  and  $\eta'$  since the  $U(1)_A$ -breaking due to the fact that the anomaly is not present at this order. Therefore we cannot directly calculate the relevant  $\eta$  amplitude as we did for  $\gamma\gamma \rightarrow \pi^0\pi^0$ . We could resort to calculate the  $\eta$  decay at tree-level to all orders in CHPT leading in  $1/N_c$  if we could obtain the relevant counterterms allowed by the symmetry



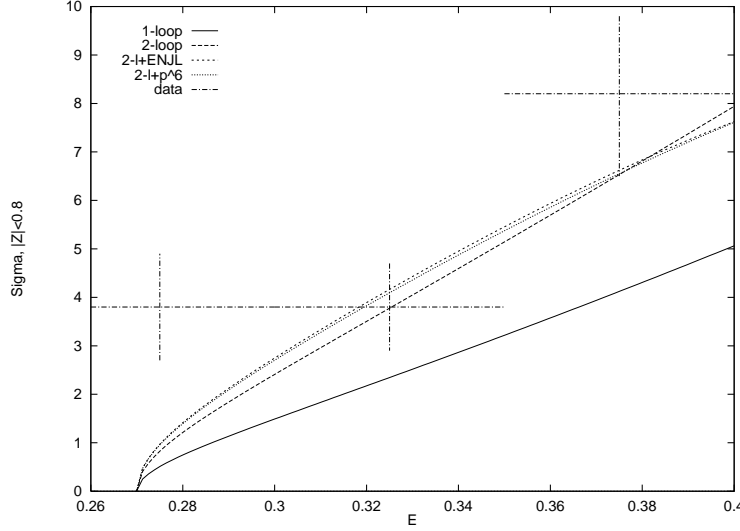


Figure 3: The cross-section for  $\gamma\gamma \rightarrow \pi^0\pi^0$  with  $|\cos\theta| \leq 0.8$ . Plotted are the result to order  $p^4$ ; the result to order  $p^6$  adding only the two-loop result, the result to the same order adding also the full ENJL result or adding only the order  $p^6$  part.

from our all orders calculation of  $\gamma\gamma \rightarrow \pi^0\pi^0$ . The physical  $\eta$  field in terms of the nonet  $\eta_8$  and singlet  $\eta_1$  SU(3) flavour states is  $\eta = \cos\varphi_P \eta_8 - \sin\varphi_P \eta_1$ . We have used  $\sin\varphi_P = -1/3$  [24]. Using the same basis as in Eq. (2) but substituting  $p_1 = p_\eta$  and  $p_2 = -p_\pi$  everywhere, the amplitude  $\eta \rightarrow \pi^0\gamma\gamma$  can be written in terms of  $A(s, \nu)$  and  $B(s, \nu)$  which to order  $p^6$  and large  $N_c$  to be consistent with our calculation of the amplitudes  $A(s, \nu)$  and  $B(s, \nu)$ , can be parametrized as:

$$\begin{aligned} A^{(6)}(s, \nu) &= \frac{4}{3f_\pi^4} \sqrt{\frac{2}{3}} \left[ 8m_\pi^2(2d_3^r - d_2^r) - 8m_\eta^2 d_2^r + (d_1^r + 8d_2^r)s \right]; \\ B^{(6)}(s, \nu) &= -\frac{2}{3f_\pi^4} \sqrt{\frac{2}{3}} d_1^r. \end{aligned} \quad (10)$$

Here,  $d_1^r$ ,  $d_2^r$  and  $d_3^r$  are the order  $p^6$  couplings of the effective chiral Lagrangian defined in Eq. (11) of [19]<sup>1</sup>. They are order  $N_c^2$  in the large  $N_c$ . These couplings also coincide with the ones defined in [8] when restricted to the two flavour case. In general (next-to leading in  $1/N_c$ ) there are three more couplings [12] which are order  $N_c$ . Only two of them appear in the amplitudes  $A(s, \nu)$  and  $B(s, \nu)$  to order  $p^6$  and can be seen as  $1/N_c$  corrections of the  $d_1^r$  and  $d_2^r$  couplings in Eq. (10) [12]. These same  $d_{1,2,3}^r$  couplings enter in the order  $p^6$  expression for

<sup>1</sup>Notice we disagree with the result for  $A^{(6)}(s, \nu)$  in that reference.

the amplitudes  $A(s, \nu)$  and  $B(s, \nu)$  for  $\gamma\gamma \rightarrow \pi^0\pi^0$ , see [8, 19]. The fact that they appear there in three different combinations allows us to disentangle them completely from the different fits to the ENJL data shown before. To obtain all counterterms contributing to the  $\eta$  decay from  $\gamma\gamma \rightarrow \pi^0\pi^0$  at higher orders is not possible. The underlying problem is easy to understand. Due to relations like  $2q_1 \cdot q_2 = 2p_1 \cdot p_2 + p_1^2 + p_2^2$ , the combinations of counterterms appearing in the amplitudes for the  $\eta$  decay and  $\gamma\gamma \rightarrow \pi^0\pi^0$  are clearly different. Therefore the only possibility to make a prediction to all orders for the  $\eta$  decay is to determine all the couplings modulating the needed counterterms. However, a quick counting shows that the number of different combinations of counterterms appearing at higher order ( $p^8$  and higher) does not allow to determine all possible terms in the chiral Lagrangian. It is enough however, as said before, for the order  $p^6$  counterterms. One could hope that going to the off-shell parts of the four-point function in (4) it could be done, but this is not the case. There are again terms that contribute to  $\gamma\gamma \rightarrow \pi^0\pi^0$  off-shell differently as to the  $\eta$  decay and terms that contribute to (4) but not to the decays. An example of the latter is

$$\text{tr} \left( F_{\mu\nu} F^{\mu\nu} \chi \chi^\dagger \right) \quad \text{with} \quad \chi = 2B_0(s + ip). \quad (11)$$

where  $s$  and  $p$  are scalar and pseudoscalar external sources as in (1). And, of course, the number of different combinations at order  $p^8$  and higher is not enough to disentangle all possible terms in the chiral Lagrangian. For this reason we will only use the order  $p^6$  part of the calculation for the  $\eta$ -decay.

From Refs. [8, 19] one can obtain the relation between the couplings  $a_1$ ,  $a_2$  and  $b_1$  in (6) and the  $d_{1,2,3}$  couplings of the order  $p^6$  Lagrangian,

$$d_1 = -\frac{9}{10} \frac{1}{(16\pi^2)^2} b_1 \quad (12)$$

$$d_2 = \frac{9}{160} \frac{1}{(16\pi^2)^2} [a_2 + 2b_1] \quad (13)$$

$$d_3 = \frac{9}{320} \frac{1}{(16\pi^2)^2} [a_1 + 2a_2 + 4b_1]. \quad (14)$$

From our ENJL results in the second column of Table (1) we get

$$d_1^r = -6.0 \cdot 10^{-5}; \quad d_2^r = 3.9 \cdot 10^{-5}; \quad d_3^r = 1.3 \cdot 10^{-5}. \quad (15)$$

We observe that the three couplings are of the same order of magnitude in this ENJL model. Notice that at order  $p^6$  there is nonet symmetry and the difference between isospin one and isospin zero is higher order.

One can also use resonance exchange dominance in the eta decay to predict the  $d_i^r$  couplings. This has been done before in [9] for the  $d_{1,2}^r$  and in [25] to predict also  $d_3^r$  using  $a_0(980)$  data. Since the actual data on the  $a_0(980)$  [24] do not allow to make any trustable prediction, we have used nonet symmetry to

obtain the  $d_3^r$  coupling from the fourth column in Table 1 with the formulas in (12). We get

$$d_1^r = -(8.2 \pm 2.0) \cdot 10^{-5}; \quad d_2^r = (4.3 \pm 1.0) \cdot 10^{-5}; \quad d_3^r = (0.4 \pm 2.7) \cdot 10^{-5}. \quad (16)$$

The  $d_i^r$  couplings obtained in (16) contain higher than order  $p^6$  corrections due to quark masses contributions to the masses of the resonances as well as next-to-leading in  $1/N_c$  corrections. Keeping this in mind, we observe that only  $d_2^r$  is in complete agreement although both results are compatible at the one  $\sigma$  level. Notice the large error bars in this way of estimating the  $d_i^r$  couplings.

The decay rate for  $\eta \rightarrow \pi^0 \gamma \gamma$  can be written as follows

$$\Gamma(\eta \rightarrow \pi^0 \gamma \gamma) = \frac{\alpha^2}{64\pi m_\eta^3} \int_0^{s_2} ds s^2 \int_{t_1}^{t_2} dt \left[ |H_{++}(s, \nu)|^2 + |H_{+-}(s, \nu)|^2 \right] \quad (17)$$

with  $\alpha$  the fine structure constant and

$$\begin{aligned} s_2 &= (m_\eta - m_\pi)^2, \\ t_{2,1} &= \frac{1}{2} \left[ m_\eta^2 + m_\pi^2 - s \pm \sqrt{(m_\eta^2 + m_\pi^2 - s)^2 - 4m_\eta^2 m_\pi^2} \right] \end{aligned} \quad (18)$$

and

$$\begin{aligned} H_{++}(s, \nu) &= A(s, \nu) + 2B(s, \nu) (2m_\pi^2 + 2m_\eta^2 - s); \\ H_{+-}(s, \nu) &= 8B(s, \nu) \frac{m_\pi^2 m_\eta^2 - ut}{s}. \end{aligned} \quad (19)$$

The experimental value of the decay rate  $\Gamma(\eta \rightarrow \pi^0 \gamma \gamma)$  is

$$\Gamma(\eta \rightarrow \pi^0 \gamma \gamma) = (0.85 \pm 0.19) \text{ eV} [24]. \quad (20)$$

Using only the order  $p^6$  tree-level contributions to  $A(s, \nu)$  and  $B(s, \nu)$  in (10) we get

$$\Gamma(\eta \rightarrow \pi^0 \gamma \gamma) = 0.18 \text{ eV} \quad (21)$$

using our ENJL results for the  $d_i^r$  couplings in (15) and the chiral limit ENJL value for  $f_\pi$ , i.e.  $f_\pi = 88.9 \text{ MeV}$ . The contribution of the coupling  $d_3^r$  is not dominant and its ENJL value results in a decreasing of the the decay rate with respect to the case with  $d_3^r = 0$ . If one instead uses the values in (16) then

$$\Gamma(\eta \rightarrow \pi^0 \gamma \gamma) = (0.18_{-0.10}^{+0.15}) \text{ eV}. \quad (22)$$

Notice that the variation within the allowed range of values for  $d_{1,2,3}^r$  predicted by the resonance exchange model produces a large uncertainty in the decay rate. This uncertainty is avoided in the ENJL model predictions.

The contribution from the order  $p^4$  loops is either suppressed by G-parity or the kaon mass [9]. The analysis in [12] of the order  $p^6$  loop contributions, though

partial, shows the same suppression. The order  $p^6$  contributions included there, which are expected to be the dominant ones at that order, interfere destructively decreasing the decay rate but only by 0.04 eV. At order  $p^8$  there appears qualitatively new contributions [9], they are the doubly-anomalous contributions. Its relative size compared to the chiral loop contributions analysed previously, cannot be inferred from the chiral counting since it is the first in its class of contributions. There is, for instance, no G-parity suppression in the couplings [9].

So, adding the order  $p^4$  charged pion and kaon loop contributions plus the doubly-anomalous contributions of order  $p^8$  and the tree-level order  $p^6$  contributions to  $A(s, \nu)$  and  $B(s, \nu)$ , we get

$$\Gamma(\eta \rightarrow \pi^0 \gamma \gamma) = 0.30 \text{ eV} \quad (23)$$

using our large  $N_c$  ENJL results for the  $d_i^r$  couplings in (15). If we instead use the values in (16) one obtains

$$\Gamma(\eta \rightarrow \pi^0 \gamma \gamma) = (0.27_{-0.07}^{+0.18}) \text{ eV}. \quad (24)$$

There is a very strong constructive interference between the order  $p^6$  tree-level and the order  $p^4$  and  $p^8$  loop contributions. Including the expected small decreasing of the order  $p^6$  loops, we conclude that within the ENJL model, the order  $p^6$  prediction for the decay rate  $\Gamma(\eta \rightarrow \pi^0 \gamma \gamma)$  is off the experimental result by almost three  $\sigma$ s. This is in disagreement with the results in [19].

In the present work, we have computed the tree-level contributions to all orders in the chiral expansion and leading in  $1/N_c$  for  $\gamma\gamma \rightarrow \pi^0\pi^0$  within the ENJL model. We have predicted the corresponding cross-section and compared with experiment. Our result shows that tree-level contributions of order higher than  $p^6$  are negligible for  $s$  and  $\nu$  below 0.2 GeV<sup>2</sup>. We have also predicted the order  $p^6$  counterterms that contribute at large  $N_c$  to this process. A comparison with other estimates of these counterterms is made. We have seen that the resonance exchange dominance works within the ENJL model to 15~25%. For the  $\eta \rightarrow \pi^0 \gamma \gamma$  we have made a prediction including the dominant chiral loop corrections [9, 12], i.e those of order  $p^4$  and the doubly-anomalous of order  $p^8$ , and the tree-level order  $p^6$  obtained within the ENJL model at leading order in  $1/N_c$ . We obtain a three  $\sigma$ s discrepancy with the experimental result previously observed in other estimates. We do not expect to obtain unusually large corrections for  $A$  and  $B$  from higher order terms because of the CHPT counting. This is the case, for instance, for the order  $p^6$  loops analysed in [12]. It should be noticed, however, that a small change in the values of  $A$  and  $B$  can result in a large enhancement of the decay rate. An example is the enhancement due to the strong constructive interference between the leading loop correction and the tree-level contributions. This is also well illustrated by the increasing of 0.14 eV (almost one  $\sigma$ ) when higher order tree-level contributions are taken into account by an “all-orders” vector meson resonance exchange model [9]. This would bring our

order  $p^6$  ENJL estimate closer to the experimental result, but still off the experimental result by not less than two  $\sigma$ s. As said in the text, the next to leading in  $1/N_c$  couplings could be regarded as corrections to  $d_1^r$  and  $d_2^r$  [12]. These have to be added to the errors inherent in our large  $N_c$  ENJL model. In view of the large constructive interference mentioned above they could add a significant contribution to the decay rate. In fact, reasonable  $1/N_c$  corrections (30 %) together with the higher orders effect above could easily bring the final result within one  $\sigma$  from the experimental result. It is then of interest to have a high statistics measurement of this decay rate and the two-photon energy spectrum in order to reduce the actual experimental uncertainty and see if this discrepancy persists. It could be also used to extract the effective  $d_i^r$  couplings and therefore deviations from our large  $N_c$  estimate. Due to its large present uncertainty, the decay rate  $\Gamma(\eta \rightarrow \pi^0 \gamma \gamma)$  can within one standard deviation be explained with higher order corrections both in  $1/N_c$  and CHPT.

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